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Some aspects of turbulence measurement in liquid mercury using cylindrical quartz-insulated hot-film sensors

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This investigation demonstrates the measurement of low turbulence intensities in a flow of mercury using the constant-temperature hot-film technique. A simple equation for the determination of turbulence intensity is derived on the basis of observed calibration data for short, cylindrical, quartz-insulated, platinum hot-film sensors. This equation has the advantage of being independent of the changeable effects at the sensor-fluid interface and of the usually troublesome drift in anemometer output signal. The flow speeds considered are low, so that the Péclet number based on the outside diameter of the insulated sensor is less than unity. Various problems associated with the application of hot-film anemometry to mercury flows are discussed.

1. Introduction

This investigation was prompted by the need for a reliable method of measuring magnetohydrodynamic turbulence quantitatively in low-Prandtl-number fluids such as liquid mercury. Details of the investigation and the subsequent experiments in magnetohydrodynamics are described by Malcolm (1968, 1969).

The earlier work of Sajben (1964, 1965) and Sajben & Fay (1967) provided a basis for this study. Sajben used a lacquer-insulated tungsten hot wire in the constant-current mode to measure turbulence intensity in a turbulent mercury jet subjected to an axial magnetic field. At the time the present work commenced, his work represented one of two known published accounts of quantitative turbulence measurements in liquid metals. The other was by Branover, Slyusarev & Shcherbinin (1965) who related strain-gauge measurements of fluctuating drag on a tear-drop-shaped probe immersed in a mercury flow to turbulence intensity. This technique does not seem to hold much promise, however, primarily because spurious noise and the size of the probe preclude the measurement of small-scale turbulence.

Sputtered-quartz-coated platinum-film sensors now manufactured by such firms as Thermo-Systems Inc. (St Paul, Minnesota, U.S.A.) and DISA Elektronik

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A/S (Herlev, Denmark) are more rugged than conventional hot wires and may be applied to low-temperature liquid-metal flows using conventional constant-temperature anemometers providing that certain special practical problems can be successfully dealt with.

In the case of mercury, the special problems concern, first, the unavoidable tendency for the hot-film sensor to be surrounded by a non-wetting layer of impurities which changes in properties with each re-immersion of the sensor and with the passage of time during any one immersion. This impurity layer creates a thermal contact resistance. Secondly, our understanding of the heat transfer from the sensors in a steady flow of a very low Prandtl-number fluid is imperfect. Finally, the response of the sensor to fluctuating heat transfer in a turbulent flow of such a fluid is not known theoretically.

The first problem can be dealt with using the calibration technique of Sajben (1965) and the turbulence measurement technique to be described. Hoff (1968) has succeeded in improving upon Sajben's calibration technique by vapour depositing a thin layer of copper or gold on the sensor's quartz insulation so that the mercury can wet the sensor.

The sensor shape under consideration is a finite-length cylinder with $L/d \simeq 16$. The heat transfer from such a sensor in a fluid for which the Prandtl number $P \simeq 0{\cdot}025$ and the Péclet number $P\acute{e} \leqslant 1$ (based upon the sensor's outside diameter) is extremely complicated. At the lowest value of $P\acute{e}$ in the calibration studies, $P\acute{e} \simeq 0{\cdot}01$, the forced convection will probably have little influence compared to natural convection, since $G/R^2 \simeq 0{\cdot}6$, where G and R are the Grashof and Reynolds numbers respectively, and secondly to the three-dimensional conduction by which heat can be transferred even if the surrounding fluid is at rest (Cole & Roshko 1954). The heat transfer from the actual sensor is further complicated by conductive end losses to the supports, a non-uniform temperature distribution, conduction through a quartz insulating layer of generally unknown thickness, and the poorly understood thermal contact resistance of the non-wetting layer which surrounds the sensor. Accordingly, only for a particular sensor during a particular immersion in mercury and for $P\acute{e} > 0{\cdot}1$ can it be said with any surety that the Nusselt number Nu is a function of $P\acute{e}$ only. Thus, no theoretical solutions for $Nu(P\acute{e})$ can be used to compare with the calibration results (e.g. the theory of Cole & Roshko (1954), valid for $P\acute{e} \rightarrow 0$, and the theory of Grosh & Cess (1958), valid for $P \ll 1$ and assuming inviscid flow).

An interesting study of the effects of directional sensitivity on the heat transfer from short, cylindrical hot-film sensors in mercury has been completed by Hill (1968). The results forecast difficulties in any attempt to apply an X-array sensor to measure turbulence in two directions. Hill observed the directional sensitivity to be strongly dependent on flow speed.

In considering the dynamic response of the cylindrical hot-film sensors in unsteady flow it is assumed that the sensor is heated by a feed-back circuit possessing a gain sufficiently high to keep the hot film at a constant mean temperature throughout the frequency range of interest, i.e. up to 10 kHz. The important points to consider are: first, unsteady heat transfer between the film and its substrate material because of thermal feed back from the fluid to the substrate; and secondly, unsteady heat transfer between the film and the surrounding fluid.

Bellhouse & Schultz (1967) have discussed the first aspect. Their results indicate that the thermal feed-back effects in the case of wedge-type sensors are serious in air but not in such fluids as water and mercury which have lower thermal impedances. These effects are expected to be negligible for cylindrical sensors in any fluid providing that the film surrounds the cylindrical quartz substrate for a length, L, such that $L/d \gg 1$ because direct feed back from the fluid to the substrate can only occur at the ends of the film. For practical sensors, where $L/d < 20$, the thermal feed back at the ends may be noticeable in air but not in mercury.

The unsteady heat transfer between the film and the fluid involves the quartz insulation which is sandwiched between them. This layer increases the sensor diameter ($d \simeq 0{\cdot}03$ mm) by 10–20 %. If one approximates the quartz insulation by an infinitely wide slab of similar material and thickness, the transient effect of a temperature jump on one side will decay in a time of $O(10^{-5})$ s. The effect of the quartz insulation on the dynamic response may therefore be neglected.

The nature of the unsteady heat transfer to the fluid has not been examined in detail theoretically for low Prandtl number. The significance of a very low Prandtl number is that the influence of the cylinder on the temperature of the fluid stretches further than its influence on the velocity. The fluid may possess considerable thermal inertia which will inhibit the response of the sensor to changes in free-stream velocity. The conventional square-wave technique of measuring sensor response, which should give a greater than actual response time in the case of a thin platinum film deposited on a quartz cylinder, has been applied by Malcolm (1968) to the 0·03 mm diameter cylindrical hot-film sensors and has shown the response time to be less than 10^{-4} s. It is therefore reasonable to expect the constant-temperature hot films to respond to turbulent fluctuations up to a frequency of at least 10 kHz.

2. Sajben's calibration equation

Let the hot-film sensor be approximated by a laminated cylinder which is long enough for end effects to be neglected. The thin lamination representing the platinum film is kept at constant temperature. The outermost 'lamination' is the non-wetting, thermal-contact-resistance layer of unknown properties and thickness. The basic heat-transfer equation is simply that given in many heat-transfer text-books for steady radial heat conduction in a laminated cylinder combined with convective heat transfer at its surface. This equation may be arranged to give

$$\frac{\pi k_f L \Delta T}{Q} = \frac{1}{Nu} + K, \tag{1}$$

where k_f is the thermal conductivity of mercury, L is the film length, ΔT is the temperature difference between the film and the mercury, Q is the ohmic dissipation in the film, Nu is the Nusselt number based upon the outside diameter of the sensor, and K is a velocity-independent term which depends upon the physical properties of both the quartz insulation and the thermal-contact-resistance layer. Sajben (1965) found experimentally that the effects of large changes in K could be eliminated by the calibration procedure:

$$F(P\acute{e}) = \pi k_f L\Delta T\left(\frac{1}{Q(0)} - \frac{1}{Q(P\acute{e})}\right) = \frac{1}{Nu(0)} - \frac{1}{Nu(P\acute{e})}, \tag{2}$$

where $Q(0)$ is the free-convective heat transfer at no flow. The calibration function F has been written as a function of $P\acute{e}$ alone for a particular sensor, although, as $P\acute{e} \to 0$, F must also depend on G. The justification for neglecting G as a variable is that the experiments to be described show F curves for a particular sensor to be independent of ΔT when ΔT is $O(10\,^{\circ}\mathrm{C})$. This phenomenon may possibly be due to the fact that, since $GP = O(10^{-3})$, $Nu(0)$ is a very weak function of G and appears to be constant over a small range of ΔT.

In the range $P\acute{e} < 1$, errors in F, as calculated from equation (2), are observed to be very sensitive to slight errors in measuring ΔT. These latter errors may arise because of a drift in environmental temperature which may be as small as $O(10^{-1}\,^{\circ}\mathrm{C/hr})$. This important aspect is discussed in the appendix. It is found that a small relative error in ΔT may produce a relative error in F which is at least ten times greater. The use of temperature-compensation equipment to complement a conventional constant-temperature anemometer is therefore advisable unless the fluid temperature is controlled to within $10^{-1}\,^{\circ}\mathrm{C}$.

3. The measurement of low-intensity turbulence

The experiments will show that the calibration equation (2) can be written in the following form for $0{\cdot}3 \leqslant P\acute{e} \leqslant 1{\cdot}0$:

$$\frac{\pi k_f L\Delta T}{Q(P\acute{e})} = A - S\ln P\acute{e}, \tag{3}$$

where A contains all effects which are independent of $P\acute{e}$.

Let $Q(P\acute{e})$ be replaced by the ohmic dissipation in the hot-film at a particular flow condition, $E^2R_s/(R_s+R^*)^2$. E is the voltage across both the hot-film resistance R_s, and R^* a resistance in series with R_s. R^* represents a resistance in the electronic circuit and the combined resistance of the current leads and the probe body. If fluid properties are considered constant, the only variable part of $P\acute{e}$ is the velocity component U perpendicular to the sensor. Equation (3) then becomes

$$\pi k_f L\Delta T(R_s + R^*)^2/E^2R_s = A' - S\ln U, \tag{4}$$

where A' includes all velocity-independent terms. Differentiating equation (4) gives

$$\frac{2}{S}\left(\frac{\pi k_f L\Delta T(R_s+R^*)^2}{R_s}\right)\frac{dE}{E^3} = \frac{dU}{U}. \tag{5}$$

It is assumed that the turbulent velocity components perpendicular to the sensor, u' in the streamwise direction and v', are very small compared to the mean velocity $\bar{u}$. It follows that $dU \simeq u'$ and $U \simeq \bar{u}+u'$ (see figure 1). The following substitutions are now made in equation (5):

$$E = \bar{E}+e, \quad dE = e, \quad U = \bar{u}+u', \quad dU = u', \tag{6}$$

where e is the small deviation from the mean anemometer output voltage $\bar{E}$ caused by the turbulent velocity fluctuation u'. This substitution is followed by a conventional root-mean-square operation in which fluctuation quantities of greater order than two are neglected, yielding the desired relationship

$$\frac{\sqrt{\overline{u'^2}}}{\bar{u}} = \frac{2}{S}\left(\frac{\pi k_f L \Delta T (R_s + R^*)^2}{R_s}\right)\frac{\sqrt{\overline{e^2}}}{\bar{E}^3}. \tag{7}$$

Equation (7) possesses the important advantage of being independent of any reference condition, such as $Q(0)$, and independent of the changeable properties of the contact-resistance layer, so that the calculation of turbulence intensity is independent of signal drift, provided that the drift is on a slower time scale than the time necessary to read a particular pair of values, $\sqrt{\overline{e^2}}$ and $\bar{E}$. Since serious signal drift is very difficult to eliminate, equation (7) would be expected to give more accurate results than more sophisticated procedures involving the linearization of the calibration curve.

The important questions to be answered experimentally are:

(i) Will equation (7) give the same value for $\sqrt{\overline{u'^2}}/\bar{u}$ at different levels of operating temperature and ohmic dissipation for an arbitrary flow condition in the range $0{\cdot}3 \leqslant P\acute{e} \leqslant 1{\cdot}0$?

(ii) Are values of $\sqrt{\overline{u'^2}}/\bar{u}$ from equation (7) actually independent of the value of A in equation (3) and therefore independent of the thickness and physical properties of the contact-resistance layer on the sensor?

(iii) Are values of $\sqrt{\overline{u'^2}}/\bar{u}$ from equation (7) actually independent of the signal drift caused by the combined effects of normal small environmental temperature changes and slow timewise variations in the properties of the contact-resistance layer?

4. Experiment

4.1. *Apparatus and methods*

A mercury-tow-tank facility was constructed for the purpose of calibrating hot-film probes in mercury; refer to Malcolm (1968) for details of its construction and operation. It is capable of speed regulation from 0·3 to 14 cm/s with ± 0·5 % precision. The vibration in the mechanism is low enough to produce an equivalent turbulence intensity of about 0·015.

A low-intensity-turbulence field was created by mounting a conventional square-mesh grid of round rods on the trolley ahead of the hot-film sensor and towing the whole assembly along the tow tank at constant speed. The grid was composed of 0·071 cm diameter brass rods in a brass frame (sprayed with PVC paint to prevent amalgamation) with a mesh length to rod diameter ratio of 5·36 and was situated at 20-mesh lengths from the sensor.

Widely varying properties of the contact-resistance layer around the sensor were obtained by repeated re-immersion through each of two types of interface, from air to mercury and from water to mercury. The water–mercury interface was obtained by pouring a layer of water on top of the mercury in the tow tank. As will be discussed in the experimental results, the differences in the heat-transfer characteristics when passing through these two different interfaces are quite dramatic. Theoretically, the turbulence intensity as calculated from equation (7) should be unaffected however. This aspect is important from a

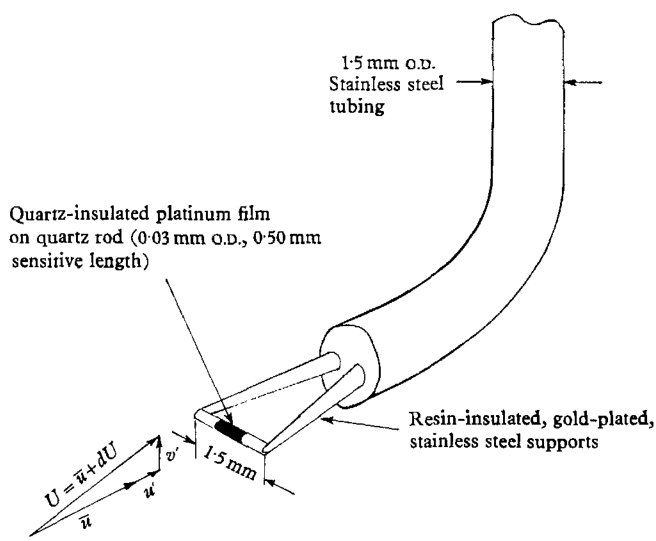


FIGURE 1. Sketch of Thermo-Systems cylindrical quartz-coated hot-film sensor for work in mercury.

practical point of view also; it is often desirable in experiments to minimize the amount of poisonous mercury vapour above free surfaces of mercury by covering them with a layer of water.

The mercury surface was cleaned periodically using a vacuum skimming device. The sensor itself was not cleaned during the course of these experiments.

No control was exerted over the temperature of the mercury or the temperature of the surroundings.

Figure 1 shows a sketch of the hot-film sensor specially manufactured by Thermo-Systems Inc. for work in mercury. It is 0·031 mm in diameter and has a sensitive length of 0·495 mm. The platinum film and the sputtered-quartz coating have thicknesses, as stated by the manufacturer, of 0·1 and 1·6 μ respectively. The support needles are insulated by a coating of resin cement. The temperature coefficient of resistance was found experimentally to be 0·00236 °C^{-1}.

The hot-film probe was operated by a Thermo-Systems Model 1010 Constant-Temperature Anemometer. ΔT was varied by adjusting the hot-film operating resistance R_s on this instrument. $\bar{E}$ was read from a digital voltmeter. $\sqrt{\overline{e^2}}$ was obtained using a Hewlett Packard Model H 12–3400 A true r.m.s. voltmeter with frequency response down to 2 Hz and with a 20 s averaging time on the d.c.

output. The instantaneous anemometer signal and the d.c. output from the r.m.s. voltmeter were continuously followed by an oscilloscope.

4.2. *Results and discussion*

Figure 2 (*a*) shows calibration results for the hot-film sensor used in these experiments. Table 1 contains descriptive information for the various sets of data points included in the figure. These data were taken at intervals during a 3-week period and during a total run time in mercury of approximately 30 h. Although a considerable amount of scatter is evident it should be emphasized that no special attention was given to cleaning the free mercury surface except to remove the thickest scum prior to each day's runs. The sensor itself was never cleaned. Special care was purposely avoided since it was considered desirable to check the reproducibility of results under realistic operating conditions. One observes from figure 2 (*a*) that measurement precision decreases as $P\acute{e} \rightarrow 1{\cdot}0$.

The ratio $\pi k_f L\Delta T/Q(0)$ takes a different value each time the sensor crosses the free surface. The higher this ratio becomes, the greater is the insulating effect of the thermal-contact-resistance layer which surrounds the sensor.

In general, the use of the mercury–water interface lessened the contact resistance. However, the increased sensitivity thus gained by improved wetting was lost because the impurity layer on the sensor seemed unstable and gave rise to random drift effects. The most satisfactory operation was experienced while using a mercury–air interface and a ΔT of about 30 °C. The best operating temperature must be chosen as a compromise between the higher temperatures, which give increased sensitivity and greater freedom from environmental temperature drift effects, and the lower temperatures, which ensure longer sensor life and lessen uncertainties in the calculation of physical properties of mercury at the mean temperature.

The calibration results are re-plotted in figure 2 (*b*), where it is observed that, for $0{\cdot}3 \leqslant P\acute{e} \leqslant 1{\cdot}0$, $F(P\acute{e})$ is a linear function of $\ln P\acute{e}$ with a slope of $-0{\cdot}31$.

When considering the accuracy of measuring turbulence intensity, it is significant that, although discrepancies exist between the various sets of data in figure 2 (*b*), the slope of each set is very nearly the same.

On an occasion when the ambient temperature remained constant (within 0·2 °C) for a period of a few hours, the low-speed calibration shown in figure 3 was carried out. At these low speeds ($\overline{u} < 0{\cdot}9$ cm/s) $F(P\acute{e})$ is observed to vary linearly with $P\acute{e}$. Since free convection is thought to be effective at these low values of $P\acute{e}$, no significance is attached to this linear variation. The instrument can therefore be usefully employed to measure very low velocities *providing* that temperature drift effects are either absent or compensated for. On many occasions calibrations at low speed were unsuccessful because significant signal drift was instigated by a temperature difference from end to end in the tow tank of as little as 0·1 °C. These temperature-drift effects are in accordance with the conclusions drawn in the appendix.

Table 2 presents the results of experiments, using the trolley and turbulence-grid apparatus, to test the turbulence-measurement formula, equation (7). The value of S in that equation was taken from figure 2 (*b*) as 0·31.

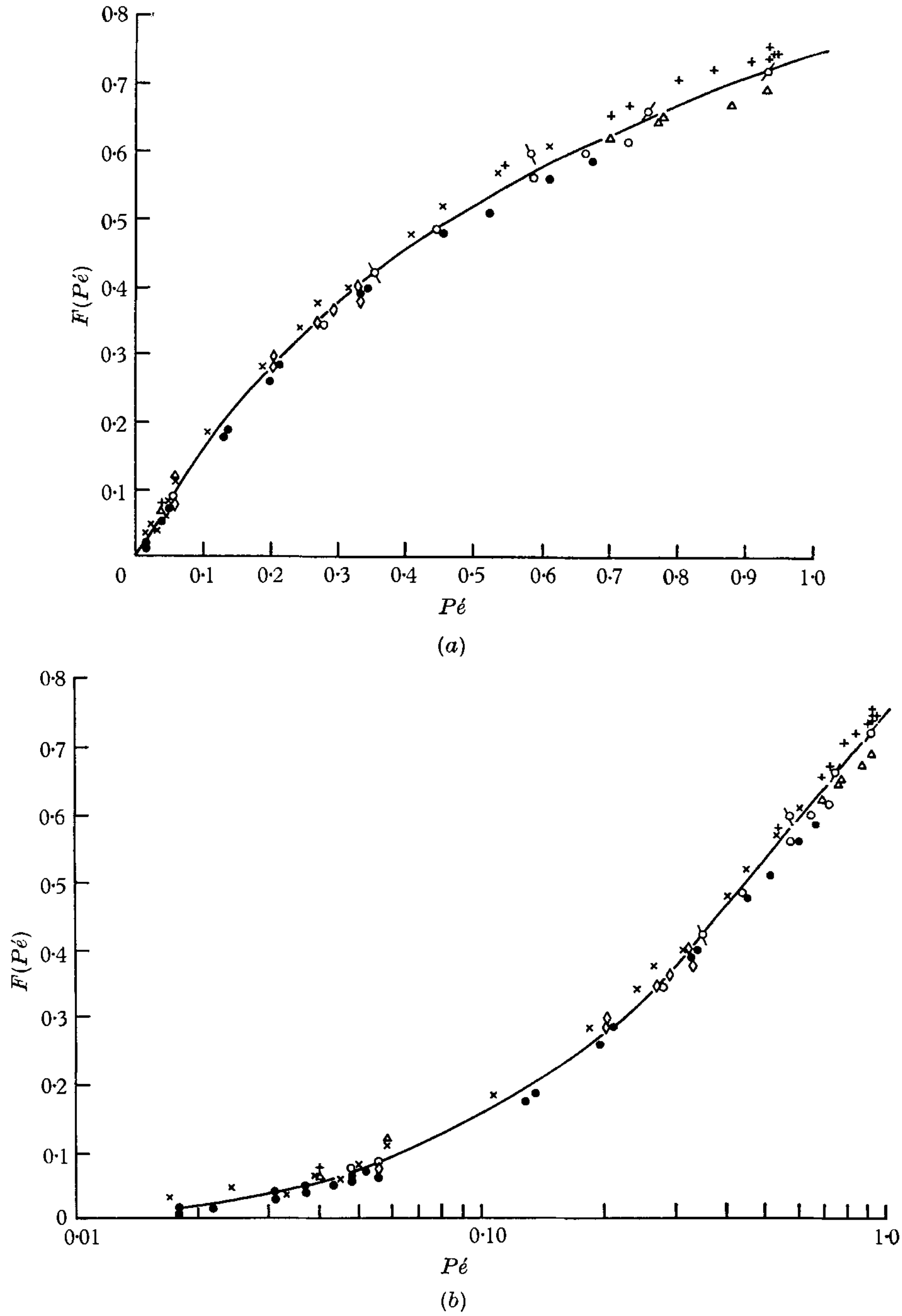


FIGURE 2. Calibration data for the Thermo-Systems hot-film sensor in mercury ($d_0 = 0{\cdot}031$ mm, $L = 0{\cdot}495$ mm). $\bar{u} = 15{\cdot}1\, Pé$ cm/sec at 25 °C. (*a*) Linear scale; (*b*) the same data on a logarithmic scale.

Symbol	$\frac{\pi k_f L \Delta T}{Q(0)}$	Interface	ΔT °C
×	6·35	Hg–air	11·4
+	8·24	Hg–air	11·3
△	4·91	Hg–air	11·3
◇	5·49	Hg–air	10·8
⌀	4·82	Hg–air	22·1
●	3·34	Clean Hg–air	33·6
◌	3·32	$Hg–H_2O$	33·0
○	3·86	$Hg–H_2O$	33·6

TABLE 1. Data for figures 2(*a*) and 2(*b*).

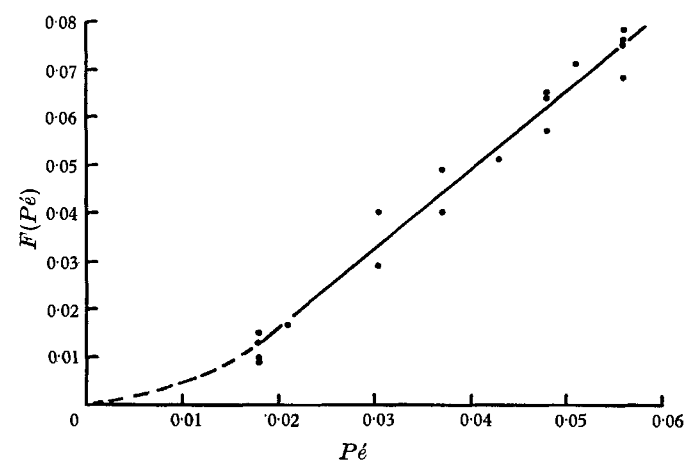


FIGURE 3. Low-speed calibration data. $(\pi k_f L\Delta T)/Q(0) = 3{\cdot}34$, $\Delta T = 33{\cdot}6$ °C, clean mercury–air interface; $\bar{u} = 15{\cdot}1$ *Pé* cm/sec at 25 °C.

In table 2 the comparison of $\bar{E}^3$ at the same ΔT yields qualitative information similar to that provided by $(\pi k_f L\Delta T)/Q(0)$ regarding contact-resistance effects. All the turbulence intensities are observed to fall in the range 0·033–0·041 and are uncorrelated with the wide variation in ΔT and the large changes in contact resistance which occur from one immersion to another through either of the two types of interface. No signal-drift effects were observed in these experiments. It is unlikely, therefore, that the variations between values of $\sqrt{\overline{u'^2}}/\bar{u}$ are due to the changing properties of the contact resistance layer. Experience with the apparatus suggests that this variation may be due to a change from one run to another in the substantial contribution to $\sqrt{\overline{u'^2}}/\bar{u}$ made by the trolley vibrations (see Malcolm 1968).

From the results of many previous researches, one would expect the value of $\sqrt{\overline{u'^2}}/\bar{u}$ in grid turbulence alone, at a position 20 mesh lengths downstream from the grid, to be about 0·028. The values obtained for the trolley–grid combination are thus quite reasonable.

The dramatic effects of a substantial improvement in thermal contact are best appreciated by comparing the results in the first two entries in table 2 with their related oscilloscope traces in figures 4(*a*) and 4(*b*). These traces have identical time and voltage scales and show typical variation of e with time for a mercury–air interface with $(\pi k_f L\Delta T)/Q(0) = 7{\cdot}2$ and for a mercury–water interface with

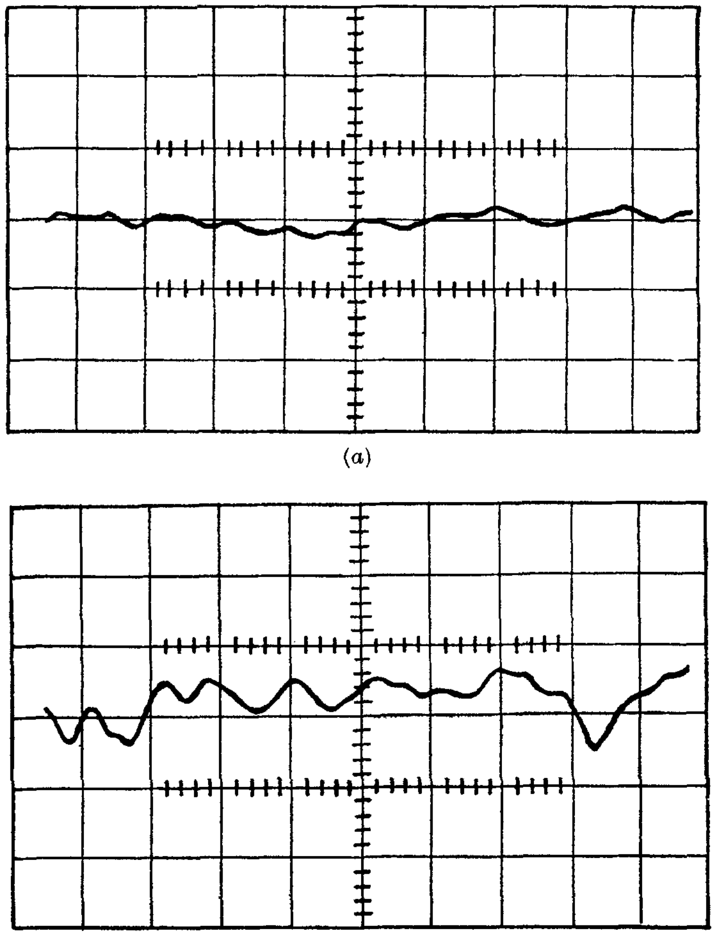
Interface	$\dfrac{\pi k_f L\Delta T}{Q(0)}$	$\bar{E}^3$	ΔT °C	$\sqrt{\overline{u'^2}}/\bar{u}$	Grid Reynolds number
Hg–air	7·2	104	33·5	0·041†	4650
Hg–H_2O	3·3	404	33·5	0·041‡	
	3·3	408	33·5	0·040‡	
Hg–air	—	48·6	10·8	0·035	3050
	—	157	22·2	0·035	
	—	290	33·1	0·034	
	—	486	43·8	0·034	
	—	185	32·8	0·036	
	—	187	32·8	0·039	
	—	319	33·0	0·035	
	—	320	33·0	0·033	
Hg–H_2O	—	65·4	10·8	0·038	
	—	372	33·1	0·036	
	—	591	43·8	0·037	
	—	370	33·0	0·040	
	—	370	33·0	0·039	
	—	362	33·0	0·041	
	—	366	33·0	0·040	
	—	346	33·0	0·038	
	—	346	33·0	0·038	

† See figure 4(*a*). ‡ See figure 4(*b*).

TABLE 2. Turbulence measurements using the trolley and turbulence–grid apparatus (horizontal lines indicate re-immersion of the probe).

$(\pi k_f L\Delta T)/Q(0) = 3{\cdot}3$, respectively. The oscillation amplitudes are much greater in the case of the mercury–water interface. This is because, at constant ΔT $\sqrt{\overline{e^2}}$ must always increase in direct proportion to $\bar{E}^3$ to yield similar values of $\sqrt{\overline{u'^2}}/\bar{u}$.

I would like to thank Prof. J. A. Shercliff for originally suggesting this work, Dr C. J. N. Alty for many helpful suggestions and Mr A. E. Webb for his skilful construction of the apparatus. In particular, I would like to thank Mr P. Bradshaw at the National Physical Laboratory and Dr D. L. Scultz at the Engineering Laboratory, University of Oxford, for very helpful discussions on various aspects of constant-temperature anemometry. This work has been made possible by an Overseas Scholarship from the Royal Commission for the Exhibition of 1851. The anemometry equipment was purchased under a Royal Society grant.



(*a*)

(*b*)

FIGURE 4. Oscilloscope traces of fluctuating voltage e. Scales: horizontal, 20 ms/div; vertical, 50 mv/div. (*a*) Mercury–air interface; (*b*) mercury–water interface.

Appendix. The effect of environmental temperature drift on measurement accuracy

A calibration procedure for hot-film measurement of flow velocity in mercury was presented in the text by equation (2):

$$F(P\acute{e}) = \pi k_f L \Delta T \left(\frac{1}{Q(0)} - \frac{1}{Q(P\acute{e})}\right) = \frac{1}{Nu(0)} - \frac{1}{Nu(P\acute{e})}. \qquad (2)$$

It is quite possible that after a short time ΔT will have drifted by 1 % due to environmental temperature changes. If a low ΔT is applied $O(10\,°C)$ to encourage long sensor life and minimize free convection, a 1 % change will be $O(10^{-1}\,°C)$ and near the limit of accuracy of ordinary temperature measurement. If such a small change is apt to go unnoticed, it is important to observe its effect on the value of $F(P\acute{e})$ in equation (2).

As explained earlier, $Nu(0)$ is a weak function of ΔT so that very small changes in ΔT will not affect $1/Nu(0)$ significantly. From equation (1) in the text it is seen that

$$\frac{1}{Nu(0)} = \frac{\pi k_f L \Delta T}{Q(0)} - K,$$

and consequently

$$\frac{\Delta T_2}{Q_2(0)} \simeq \frac{\Delta T_1}{Q_1(0)}, \tag{A 1}$$

where ΔT_2 is slightly different from ΔT_1 because of temperature drift effects. Combining equations (2) and (A 1) yields the true value $F_t(P\acute{e})$, at either ΔT_1 or ΔT_2,

$$F_t(P\acute{e}) = \pi k_f L\left(\frac{\Delta T_1}{Q_1(0)} - \frac{\Delta T_1}{Q_1(P\acute{e})}\right) \simeq \pi k_f L\left(\frac{\Delta T_1}{Q_1(0)} - \frac{\Delta T_2}{Q_2(P\acute{e})}\right). \tag{A 2}$$

An error will arise on the right-hand side of equation (A 2) if ΔT_2 is mistakenly thought by the observer to have remained at ΔT_1. In this case, the apparent value $F_a(P\acute{e})$ is

$$\left.\begin{aligned} F_a(P\acute{e}) &= \pi k_f L\left(\frac{\Delta T_1}{Q_1(0)} - \frac{\Delta T_1}{Q_2(P\acute{e})}\right), \\ \text{or} \qquad F_a(P\acute{e}) &= \pi k_f L\left(\frac{\Delta T_1}{Q_1(0)} - \frac{\Delta T_2}{Q_2(P\acute{e})}\frac{\Delta T_1}{\Delta T_2}\right). \end{aligned}\right\} \tag{A 3}$$

Let the respective relative errors in ΔT and $F(P\acute{e})$ be defined as

$$\epsilon_T = \frac{\Delta T_2 - \Delta T_1}{\Delta T_1}, \quad \epsilon_F = \frac{F_a - F_t}{F_t}. \tag{A 4}$$

In order to compare the values of the terms in equation (A 3), let the following ratio be defined at any $P\acute{e}$:

$$\left[\frac{\Delta T_1}{Q_1(0)} - \frac{\Delta T_2}{Q_2(P\acute{e})}\right] \Big/ \left(\frac{\Delta T_1}{Q_1(0)}\right) = z. \tag{A 5}$$

Combining equations (A 2) to (A 5) results in the error equation

$$\epsilon_F = \left(\frac{1}{z} - 1\right)\left(1 - \frac{1}{\epsilon_T + 1}\right). \tag{A 6}$$

From equation (A 6) it may be noted that an increase in ϵ_T or a decrease in z brings about an increase in ϵ_F. The value of z at a particular $P\acute{e}$ depends upon the properties of the thermal-contact-resistance layer surrounding the sensor. In general z increases as the contact resistance decreases. Suppose that, at a certain $P\acute{e}$, z takes a typical value of 0·02. If ϵ_T is assigned a typical value of 0·005, it is easily calculated that $\epsilon_F = 0{\cdot}24$. Thus a 0·5 % error in ΔT has brought about a 24 % error in $F(P\acute{e})$ which, when referred to the calibration curve, gives a correspondingly large error in $P\acute{e}$. As a consequence of this error propagation, it is advisable either to control the environmental temperature accurately or else to employ an electronic temperature compensator which follows that temperature continuously and makes appropriate changes in R_s to keep ΔT constant. In any case, it is advisable to run at as high a level of ΔT as is practically feasible to minimize ϵ_T.

It is evident from equation (7), which is linearly dependent on ΔT, that the relative error in turbulence intensity due to temperature drift will be equal to ϵ_T.

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